Exercise 9

Find dy/dx by implicit differentiation.

$$\frac{x^2}{x+y} = y^2 + 1$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}\left(\frac{x^2}{x+y}\right) = \frac{d}{dx}(y^2+1)$$

$$\frac{\left[\frac{d}{dx}(x^2)\right](x+y) - \left[\frac{d}{dx}(x+y)\right](x^2)}{(x+y)^2} = \frac{d}{dx}(y^2) + \frac{d}{dx}(1)$$

$$\frac{(2x)(x+y) - \left[\frac{d}{dx}(x) + \frac{d}{dx}(y)\right](x^2)}{(x+y)^2} = 2y \cdot \frac{d}{dx}(y) + (0)$$

$$\frac{(2x)(x+y) - [(1)+(y')](x^2)}{(x+y)^2} = 2yy'$$

$$\frac{2x^2 + 2xy - x^2 - x^2y'}{(x+y)^2} = 2yy'$$

Multiply both sides by $(x+y)^2$.

$$2x^2 + 2xy - x^2 - x^2y' = 2y(x+y)^2y'$$

$$x^2 + 2xy = [x^2 + 2y(x+y)^2]y'$$

Solve for y'.

$$y' = \frac{x^2 + 2xy}{x^2 + 2y(x+y)^2}$$